



Mark Scheme (Results)

January 2022

Pearson Edexcel International Advanced Level In
Mechanics M3 (WME03) Paper 01

Question Number	Scheme	Marks
1.	$\text{Area} = \int_0^a (x^2 + ax) dx = \left[\frac{1}{3}x^3 + \frac{1}{2}ax^2 \right]_0^a = \frac{5a^3}{6}$ $\int \frac{1}{2}y^2 dx = \int_0^a \frac{1}{2}(x^4 + 2ax^3 + a^2x^2) dx$ $= \frac{1}{2} \left[\frac{1}{5}x^5 + \frac{a}{2}x^4 + \frac{a^2}{3}x^3 \right]_0^a \left(= \frac{31a^5}{60} \right)$ $\bar{y} = \frac{\int \frac{1}{2}y^2 dx}{\int y dx} = \frac{31a^5}{60} \div \frac{5a^3}{6} = \frac{31a^2}{50}$	M1A1 M1 DM1A1 M1A1 (7) [7]
M1 A1 M1 DM1 A1 M1 A1	Attempt the area by integration. Powers of both terms to increase by 1. Correct area. Use $\int \frac{1}{2}y^2 dx$ to give $\int_0^a \frac{1}{2}(x^4 + 2ax^3 + a^2x^2) dx$. Limits not needed. Squaring to be correct. For method mark, condone missing $\frac{1}{2}$ or any multiple. Attempt the integration (powers of at least 2 terms to increase by 1). Depends on second M mark. Correct integration and correct limits shown. Limits needed but substitution does not need to be seen. Use $\bar{y} = \frac{\int \frac{1}{2}y^2 dx}{\int y dx}$ Note: This independent method mark is for use of the correct formula. Correct answer.	

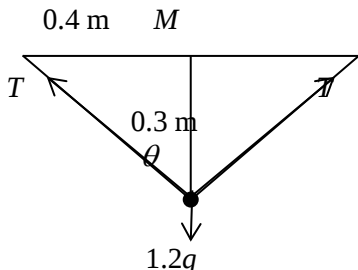
Question Number	Scheme	Marks
2	Any correct sin or cos ratio. $T \cos 60^\circ + N = mg$ $T \sin 60^\circ = mr\omega^2 = m\omega^2 \times 2l \sin 60^\circ$ $\frac{1}{2}T + N = mg \quad \frac{1}{2}T = ml\omega^2$ $\Rightarrow ml\omega^2 + N = mg$ $N \geq 0 \Rightarrow l\omega^2 \leq g$ $\omega \leq \sqrt{\frac{g}{l}} \quad *$	B1 M1A1 M1A1 DM1 DM1 A1 * (8) [8]
B1 M1 A1 M1 A1 DM1 DM1 A1*	May be seen explicitly or used in an equation. Attempt at vertical resolution, 3 terms needed. Correct equation. Attempt an equation for NL2 along the radius, acceleration in either form but not 'a'. May have r and v in the equation. Fully correct equation with the acceleration in $r\omega^2$ form and radius in terms of l. Eliminate T Depends on both M marks above. Must see an equation still involving N. Use $N \geq 0$ Depends on all 3 M marks above. Must see correct inequality stated, not $N = 0$ or $N > 0$. Reach the given result from fully correct working.	
ALT B1 M1 A1 DM1 M1 A1 DM1 A1*	For solutions that do not use vertical equilibrium but go straight to a vertical inequality. As above. Forming a correct inequality $T \cos 60 \leq mg$ Attempt NL2 as the main mark scheme. Eliminate T Depends on M marks above. Reach the given result from fully correct working.	

Question Number	Scheme	Marks
3		
(a)	$mv \frac{dv}{dx} = mg \sin \alpha - \frac{1}{3}mx^2$ $\frac{1}{2}v^2 = xg \sin \alpha - \frac{1}{9}x^3 \quad (+c)$ $x = 2 \quad \frac{1}{2}v^2 = 2g \sin \alpha - \frac{8}{9}$ $(v = 3.728\dots)$ $v = 3.7 \text{ or } 3.73 \text{ (ms}^{-1}\text{)}$	M1A1 DM1A1 DM1 A1cso (6)
ALT	By energy: $mg \sin \alpha x = \int \frac{1}{3}mx^2 dx + \frac{1}{2}mv^2$ $xg \sin \alpha = \frac{1}{9}x^3 + \frac{1}{2}v^2 \quad (+c)$ $x = 2 \quad \frac{1}{2}v^2 = 2g \sin \alpha - \frac{8}{9}$ $v = 3.7 \text{ or } 3.73 \text{ (ms}^{-1}\text{)}$	M1A1 DM1A1 DM1 A1
(b)	$v = 0 \Rightarrow x^2 = 9g \sin \alpha = 9g \times \frac{2}{5} \quad (x \neq 0)$ $x = 5.939\dots \Rightarrow OA = 5.9 \text{ or } 5.94 \text{ (m)}$	M1A1 (2) [8]

Question Number	Scheme	Marks
(a)		
M1	Attempt an equation of motion parallel to the plane with acceleration in any form (including a)	
A1	Correct equation with the acceleration in $v \frac{dv}{dx}$ form	
	Attempt the integration, powers increase by 1 in 2 terms – the constant may be missing.	
DM1	Acceleration must be in $v \frac{dv}{dx}$ form.	
A1	Correct integration.	
DM1	Sub $x = 2$ in their expression for v^2 Depends on all previous M marks.	
A1	Correct value for v and $+c$ should be dealt with. Must be 2 or 3 sf	
ALT		
M1	Attempt a 3 term energy equation – KE, GPE, work done. Integral form is not required here.	
A1	Fully correct equation with integral form for work done. Rest as main scheme.	
(b)		
M1	Use $v = 0$ in their expression for v and obtain a value of x	
A1	Correct value of length OA. (Allow if x instead of OA) Must be 2 or 3 sf (unless already penalised in (a))	
ALT (b)		
M1	Start again with energy and integrate to obtain a value of x	
A1	See mark scheme.	

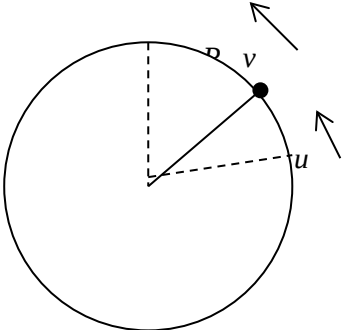
Question Number	Scheme	Marks
4(a)	Ratio of masses: $4\pi a^2$ $4\pi a \times ka$ $8\pi a^2$ $12\pi a^2 + 4k\pi a^2$	B1
	Distances : (0) $\frac{k}{2}a$ $(1+k)a$ $\bar{y}$	B1
	$(0+)k \times \frac{k}{2}a + 2(1+k)a = (k+3)\bar{y}$	M1A1ft
	$\left(\frac{k^2}{2} + 2 + 2k\right)a = (k+3)\bar{y}$	
	$\bar{y} = \frac{(k^2 + 4k + 4)}{2(k+3)}a$ *	A1 * (5)
(b)	$\tan 60^\circ = \frac{(k^2 + 4k + 4)}{2(k+3)}a \div 2a$	M1
	$k^2 + 4k(1 - \sqrt{3}) + (4 - 12\sqrt{3}) = 0$	A1
	$k > 0 \Rightarrow k = 5.8147... = 5.8$ or 5.81 or better	A1 (3)
[8]		
(a) B1 B1 M1 A1ft A1* (b) M1 A1 A1	Correct ratio of masses – any equivalent to that shown Correct distances from O or a parallel axis. Attempt a moments equation. Must be dimensionally correct (not using volumes) and have no extra terms. Correct equation, follow through their ratio of masses and distances Correct given expression with sufficient working Use $\tan 60 = \frac{\bar{y}}{2a}$ or $\frac{2a}{\bar{y}}$ May also use $\tan 30$ Obtain the correct 3TQ Correct value for k. Note for (a): The distance from O for the combined cylinder and base is $\frac{ak^2}{2(1+k)}$.	

Question Number	Scheme	Marks
5(a)	$x = 4 \cos\left(\frac{1}{5}\pi t\right) \quad \dot{x} = -4 \times \frac{\pi}{5} \sin\left(\frac{1}{5}\pi t\right)$ $\ddot{x} = -4 \times \left(\frac{\pi}{5}\right)^2 \cos\left(\frac{1}{5}\pi t\right)$ $\ddot{x} = -\left(\frac{\pi}{5}\right)^2 x \therefore \text{SHM}$	M1A1 A1 (3)
(b)	period = $\frac{2\pi}{\frac{\pi}{5}} = 10$ (s)	M1A1 (2)
(c)	amplitude = 4 (m)	B1 (1)
(d)	$\dot{x} = -4 \times \frac{\pi}{5} \sin\left(\frac{1}{5}\pi t\right) \quad \text{or} \quad \dot{x}_{\max} = a\omega$ $\text{Max speed} = 4 \times \frac{\pi}{5} = \frac{4\pi}{5} \quad \text{or} \quad 0.8\pi \text{ (ms}^{-1}\text{)}$	M1 A1 (2)
(e)	At A $x = 1.5 \quad 1.5 = 4 \cos\left(\frac{1}{5}\pi t\right) \Rightarrow t_A = \frac{5}{\pi} \cos^{-1}\left(\frac{1.5}{4}\right)$ At B $x = -2.5 \quad -2.5 = 4 \cos\left(\frac{1}{5}\pi t\right) \Rightarrow t_B = \frac{5}{\pi} \cos^{-1}\left(\frac{-2.5}{4}\right)$ Time A to B $= t_B - t_A = \frac{5}{\pi} \cos^{-1}\left(\frac{-2.5}{4}\right) - \frac{5}{\pi} \cos^{-1}\left(\frac{1.5}{4}\right) = 1.6862... = 1.7 \text{ or better (s)}$	M1A1 A1 A1 (4)
		[12]
(a) M1 A1 A1 (b) M1 A1 (c) B1 (d) M1 A1 (e) M1 A1 A1 A1	Differentiate the given expression for x twice (Both derivatives must be shown) Need to see: cos to sin to cos (ignore signs) Both derivatives correct Rewrite in the standard form for SHM and give the conclusion. Correct method Correct period Correct amplitude Use either method to obtain the max speed Correct max speed Find the time from the start to either A or B One correct time Second relevant time Correct time from A to B. 1.7 (s) or better	

Question Number	Scheme	Marks
ALT (e)	$1.5 = 4 \sin\left(\frac{1}{5}\pi t\right) \Rightarrow t_A = \frac{5}{\pi} \sin^{-1}\left(\frac{1.5}{4}\right)$ $2.5 = 4 \sin\left(\frac{1}{5}\pi t\right) \Rightarrow t_B = \frac{5}{\pi} \sin^{-1}\left(\frac{2.5}{4}\right)$ ${}_A t_B = t_B + t_A = \frac{5}{\pi} \sin^{-1}\left(\frac{2.5}{4}\right) + \frac{5}{\pi} \sin^{-1}\left(\frac{1.5}{4}\right) = 1.6862.. \text{ 1.7 or better}$	M1A1 A1 A1
6(a)	$T = \frac{\lambda x}{l} \Rightarrow 30 = \frac{\lambda \times 0.3}{0.5}$ $\lambda = 50 *$	M1A1 A1* (3)
(b)	 Extension = 0.5 m (used in (b) or (c)) $T = \frac{50 \times 0.5}{0.5} = (50)$ $R(\uparrow) \quad 2T \cos \theta - 1.2g = 1.2a$ $100 \times \frac{3}{5} - 1.2 \times 9.8 = 1.2a$ $a = 40.2 \quad a = 40 \text{ or } 40.2 \text{ m s}^{-2} \text{ (positive)}$	B1 M1A1ft M1 A1ft A1 (6)
(c)	$\text{E.P.E.} = \frac{1}{2} \times 50 \times \frac{0.5^2}{0.5}$ $1.2g \times 0.3 + \frac{1}{2} \times 1.2v^2 = \frac{1}{2} \times 50 \times \frac{0.5^2}{0.5} - \frac{1}{2} \times 50 \times \frac{0.3^2}{0.5}$ $v^2 = \frac{1}{0.6} \left(25 \times \frac{0.5^2}{0.5} - 25 \times \frac{0.3^2}{0.5} - 1.2g \times 0.3 \right) (= 7.452)$ $v = 2.730... = 2.7 \text{ or } 2.73 \text{ m s}^{-1}$	B1ft (any correct EPE) M1A1A1 DM1 A1 (6)

Question Number	Scheme	Marks
		[15]
(a) M1 A1 A1*	Use HL with $T = 30$ Correct equation Obtain given value for λ from fully correct working	
(b) B1 M1 A1ft M1 A1ft A1	Correct extension, seen explicitly or used in (b) or (c) Extension 0.25 if half string used. Use HL to form an equation using $\lambda = 50$ and their extension. Correct equation, ft their extension Attempt a vertical equation of motion. Must have 3 terms with T resolved a could be negative. Correct equation with their value of T Correct value of the acceleration (positive). Must be 2 or 3 sf.	
(c) B1ft M1	Either EPE correct, follow through their extension. Energy equation, from C to the ceiling. PE, KE and 2 EPE terms required. EPE of the form $k \frac{x^2}{l}$	
A1 A1 DM1 A1	. Both EPE terms correct. Completely correct equation Solve for v^2 or v Correct value for v . Must be 2 or 3 sf (unless penalised in (b)).	

Question Number	Scheme	Marks
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7	 (a) $mgl(\cos \theta - \cos \alpha) = \frac{1}{2}mu^2 - \frac{1}{2}mv^2$ $v^2 = u^2 - 2gl(\cos \theta - \cos \alpha)$ * (b) $\cos \alpha = \frac{2}{5} \quad v^2 = 3gl - 2gl\left(\cos \theta - \frac{2}{5}\right)$ At top $\theta = 0^\circ \quad v^2 = 3gl - 2gl \times \frac{3}{5}$ $v^2 = \frac{9gl}{5}$ $v^2 > 0 \Rightarrow$ complete circle * (c) Equation of motion along radius at lowest point: $kT - mg = \frac{mw^2}{l}$ $\theta = 180 \quad w^2 = 3gl - 2gl\left(-1 - \frac{2}{5}\right) = \frac{29gl}{5}$ $kT = \frac{m}{l} \times \frac{29gl}{5} + mg = \frac{34mg}{5}$ At highest point: $T_2 + mg = \frac{mv^2}{l}$ $\theta = 0 \quad T = \frac{9mg}{5} - mg = \frac{4mg}{5}$ $k \frac{4mg}{5} = \frac{34mg}{5} \Rightarrow k = \frac{17}{2}$	M1A1A1 A1* (4) M1 A1 M1 A1* (4) M1A1 M1 M1A1 M1 M1 A1 A1 [17]
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Question Number	Scheme	Marks
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(a)	
M1	Attempt energy equation from A to general position. Must have a difference of 2 PE terms and a difference of 2 KE terms.
A1	Correct gain in PE or loss of KE
A1	Fully correct equation
A1*	Reach the given result from fully correct working
(b)	
M1	Sub $u = \sqrt{3gl}$ and $\cos\alpha = \frac{2}{5}$ in the result in (a)
A1	Correct equation
M1	Put $\theta = 0$ to find an expression for v^2 at the top (maybe finding KE)
A1*	Fully correct working and conclusion with reason eg reference to v^2 , v , KE
(c)	
M1	Form an equation of motion along the radius at the lowest point. Acceleration in either form.
A1	Correct equation with acceleration in v^2/r form
M1	Use $\theta = 180$ in result from (a) to obtain an expression for w^2
M1	Eliminate w^2 and obtain an expression for kT
A1	Correct expression for kT
M1	Form an equation of motion along the radius at the highest point. Acceleration in either form.
M1	Sub $\theta = 0$ and obtain an expression for T
A1	Correct expression for T
A1	Correct value of k . Must be exact.
NB	The equation of motion at the top may be seen first. Award M1A1 for either equation correct and M1 for the second.

Question Number	Scheme	Marks
ALT1 7(c)	Better equation of motion at top or bottom: $T - mg = \frac{mv^2}{l}$ $T + mg = \frac{mv^2}{l}$	M1 A1
	Other equation of motion – see above	M1
	Finding speed at the bottom: $\theta = 180 \quad w^2 = 3gl - 2gl\left(-1 - \frac{2}{5}\right) = \frac{29gl}{5}$	M1
	Finding maximum Tension (lowest point) $\theta = 180, \quad T = \frac{m}{l} \times \frac{29gl}{5} + mg = \frac{34mg}{5}$	M1 A1
	Finding minimum Tension (highest point) $\theta = 0 \quad T = \frac{9mg}{5} - mg = \frac{4mg}{5}$	M1 A1
	Dividing Tensions to reach the correct answer $k \frac{4mg}{5} = \frac{34mg}{5} \Rightarrow k = \frac{17}{2}$	A1

Question Number	Scheme	Marks
ALT 2 7 (c)	General equation of motion: $T + mg\cos\theta = \frac{mv^2}{l}$	M1 A1
	Use of $u = \sqrt{3gl}$ and $\cos\alpha = \frac{2}{5}$ to replace v^2 in their equation of motion	M1
	Finding speed at the lowest point: $\theta = 180 \quad w^2 = 3gl - 2gl\left(-1 - \frac{2}{5}\right) = \frac{29gl}{5}$	M1
	Finding maximum Tension (lowest point) $\theta = 180, \quad T = \frac{m}{l} \times \frac{29gl}{5} + mg = \frac{34mg}{5}$	M1 A1
	Finding minimum Tension (highest point) $\theta = 0, \quad T = \frac{9mg}{5} - mg = \frac{4mg}{5}$	M1 A1
	Dividing Tensions to reach the correct answer $k \frac{4mg}{5} = \frac{34mg}{5} \Rightarrow k = \frac{17}{2}$	A1